The AdS/CFT correspondence and applications to QCD

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Abstract

In this dissertation, an introduction into the the subject of the AdS/CFT correspondence is presented. As a concrete example, we explore Maldacena's original conjecture of a duality between $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and Type IIB supergravity on $AdS_5 \times S^5$. The ingredients of both theories necessary for establishing the correspondence are reviewed and the connection at the level of correlation functions is explained. Several existing methods for modifying the setup in which the correspondence takes place, in order to extend it to theories more similar to QCD, are described.

Chapter 1

Introduction

Currently, the widely accepted theory of elementary particles and their interactions is the Standard Model – a gauge quantum field theory with local gauge symmetry group $SU(3) \times SU(2) \times U(1)$. The Standard Model is, however, not a complete description of nature – it incorporates three of the four fundamental interactions, leaving the description of gravity to the equations of General Relativity. Both theories have been tested to extreme precision in the domains in which they are valid; however, they prove to be irreconcilable when applied to situations invoking them at the same time. This incompatibility is the motivation behind the search for a theory of quantum gravity – one that would describe what happens, for example, during the final moments of the life of an evaporating black hole. Today, one of the most prominent candidates for such a theory is string theory. What is more, string theory can be used to predict the elementary particles of the Standard Model (and many others not included in the latter). In doing so, it contains all the necessary features of a potential Theory of Everything encompassing all fundamental forces and elementary particles.

String theory originated in the late 1960's, when it was discovered that it might be possible to describe the spectra of hadrons by modelling gluon flux tubes between quarks as strings. However, this idea led to inconsistencies such as the presence of tachyons [1, §1] and massless spin 2 particles for which there is no observational evidence. Although the gauge field theory of Quantum Chromodynamics soon proved to be a much more accurate description of the strong interaction, it was later realised that the spin 2 particles can be seen as quanta of the gravitational field and the potential of string theory as a unifying theory of all four fundamental forces was unveiled. The years until the turn of the century were marked by many technical challenges but also by two "superstring revolutions", when crucial insights were made and interest in string theory strengthened [2, chap. 6]. In particular, in the second superstring revolution of 1995, light was cast on the role of dualities and the way they establish the connection between what were previously considered five unrelated versions of string theory. The discovery of these dualities greatly enhanced the understanding of the subject and pushed the frontiers of research to new and unexplored areas such as M-theory, the conjectured unperturbed theory to which the five string theories are different approximations.

This is why when in 1997 Juan Maldacena, then at Harvard University, published a landmark paper in which he gave an example of a new, different kind of duality [3], the direction of string theory research changed dramatically once again. Maldacena's conjecture establishes the equivalence between a string theory living in 10 dimensions and a conformal field theory in 4 dimensions: it is precisely this equivalence that carries the name "the AdS/CFT correspondence", the subject of this dissertation. Specifically, the AdS/CFT correspondence is the conjectured duality between a quantum string theory living on a particular 10-dimensional geometry and a specific quantum field theory (QFT) living on the boundary of that geometry. The 10-dimensional geometry is $AdS_5 \times S^5$ and its boundary is 4-dimensional Minkowski spacetime, $\mathbb{R}^{1,3}$. The string theory living on $AdS_5 \times S^5$ is the low-energy effective theory of Type IIB superstring theory with string coupling g_s. The QFT living on $\mathbb{R}^{1,3}$ is $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with gauge group $SU(N)$ and coupling g_{YM}, and is both a gauge theory and a conformal field theory.

There are different ways to refer to the correspondence, each alluding to one of its main features. First, the equivalence is established in such a way that when one of these theories is in its strongly coupled regime, the other one is weakly coupled and vice versa. This is encapsulated by the term *duality*. The original correspondence between the two theories mentioned above is just the best studied example of a *gauge/string* duality – that is, a duality between a gauge field theory and a string theory. Since one often considers a weaker form of the duality, valid in the low-energy limit of the two theories, the name *gauge/gravity* duality is also very common, supergravity being the low-energy effective theory of superstring theory. Finally, because the correspondence identifies two theories of different dimensionalities, it is often referred to simply by *holography*, because of the ability to encode the physical information contained in degrees of freedom living in 10 dimensions into ones living in less dimensions.

One of the reasons for which the gauge/string duality is regarded as a remarkable result is that it brings string theory one step closer to the accepted theories governing particle interactions, which are gauge theories. Also, there is a considerable advantage in the fact that the correspondence is a duality, matching two theories in different coupling regimes. This is because calculations which in principle cannot be carried out in the non-perturbative regime of one theory can instead be computed on the other side, where the coupling is on the contrary small.[1] In the case of Quantum Chromodynamics (QCD), which is a strongly coupled theory at low energies, finding a string theory dual might bring new insights into its strongly coupled regime – and hence allow for a better understanding of the low-energy mechanisms of confinement and chiral symmetry breaking, for example.

[1] The disadvantage of a duality between two theories is that, unless the equivalence is proven, it is difficult to test it by computing quantities on both sides and comparing them afterwards.

It is important to note that the AdS/CFT correspondence has no mathematical proof and has the status of a conjecture. As we will see at the end of chapter 2, there are three different approximations at which the duality is assumed to hold, each one more restrictive than the previous. In the weakest and best studied form of the correspondence (the most restrictive scenario), the gauge group of the theory on the gauge side is $SU(N)$, where N is taken to be large, $N \gg 1$. In addition, in all three approximations, the gauge theory must exhibit conformal symmetry and supersymmetry. On the other hand, the gauge group of QCD is $SU(3)$, and QCD has a strictly negative β function and hence a running coupling – the strong interaction certainly changes at different scales and is not conformally invariant, and neither it is supersymmetric for that matter. Furthermore, QCD contains matter fields, whereas all the fields in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory transform in the adjoint representation of the gauge group.

Thus, given these obstacles, we arrive at the aims of this dissertation. The main purpose is to give an overview of AdS/CFT by reviewing the original example of the correspondence and investigating how exactly the two theories are matched to one another. A secondary aim is to review the existing methods of extending the AdS/CFT correspondence to QCD – in other words, to investigate how different aspects of QCD are seen to arise from a supergravity theory in the light of the gauge/gravity duality. Chapter 2 is devoted to the first of these aims and Chapter 3 to the second one.

Chapter 2

The AdS/CFT correspondence

In this chapter we will study the original and best understood example of a gauge/gravity duality, the correspondence between Type IIB supergravity living on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills theory ($\mathcal{N} = 4$ SYM) living on $\mathbb{R}^{1,3}$. The fact that the latter is a conformal field theory (CFT) gives the name AdS/CFT to this correspondence and, by extension, to the whole line of research focusing on the gauge/gravity duality. To begin, our first aim will be to understand what is meant by a duality between two theories, and this is the subject of section 2.1. Next, we will move on to discussing the theories on each side of the correspondence: in section 2.2 we will study $\mathcal{N} = 4$ SYM, the theory on the gauge side, and in section 2.3 we will investigate some aspects of Type IIB supergravity, the theory on the gravity side. Finally, we will make the connection between the two in section 2.4.

2.1 The concept of a dual theory

Before diving into the AdS/CFT correspondence, let us first explain the meaning of the statement that two theories are equivalent or otherwise said to correspond to each other. To do this, we will take a step back and explore the basic elements that a theory is composed of. We will go through the fundamental notions of a system and a state, to eventually arrive at physical observables, those special quantities through which a theory makes contact with the real world and its predictions can be tested against experiment. We will see that, in order to establish a correspondence between any two theories, it is precisely the observables of the two theories that we have to identify.

In theoretical physics, the term *theory* is a synonym for mathematical model of a physical system. A mathematical model consists of degrees of freedom, which evolve with respect to a particular domain, according to some chosen laws of physics[1]. Often, a theory will

[1] As a side note, a very important question is what are the mathematical objects that embody the degrees of freedom. In classical mechanics, these are real-valued functions of the background domain. In quantum

contain additional information such as values for particular parameters, renormalisation prescriptions, etc. At the most basic level, however, a particular theory is specified by its content (the degrees of freedom), the background space (the domain consisting of the independent variables) and a Lagrangian (the laws of physics). As an example, consider $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in flat 3+1 dimensional spacetime.

- The degrees of freedom of this theory are one gauge field in the $(\frac{1}{2},\frac{1}{2})$ representation of the Poincaré group, four fermion fields in the $(\frac{1}{2},0)$ representation, and six scalar fields in the (0,0) representation [4, §2.4]. The direct sum of these representations is a representation of the $\mathcal{N} = 4$ super-Poincaré group, the so-called vector supermultiplet.

- The space over which these fields are allowed to vary is 3+1 dimensional Minkowski spacetime. Note that this is one possible formulation of $\mathcal{N} = 4$ SYM; in other formulations the background space is taken to be an extended version of Minkowski spacetime that includes fermionic, or anticommuting, coordinates. Such a space is called a superspace and fields defined over it are called superfields. For an overview of the different formulations of $\mathcal{N} = 4$ SYM, see [5, §2.1].

- Finally, the Lagrangian of this theory is the Yang-Mills Lagrangian whose main characteristic is invariance under the $SU(N)$ gauge group. Since this is a supersymmetric version of Yang-Mills theory, the action is of course invariant under SUSY transformations. Accidentally, this theory also exhibits conformal symmetry. The full Lagrangian of $\mathcal{N} = 4$ SYM and a discussion of its symmetries is given in Section 2.2.

These are the three building blocks of a theory. Within this framework, a *system* refers to the combination of the first two: it is defined by its degrees of freedom and the background space, i.e. a set of functions over a domain. The main object of study of theoretical physics is how systems evolve throughout the background space and what is the information that can be extracted about them, especially at some subsets of the background space. This is where the notion of a *state* becomes useful. Intuitively, a state is some kind of snapshot of the system at a particular time. However, in a relativistic string-theoretic setting, where time is observer-dependent and the background space is not a one-dimensional worldline but rather a higher-dimensional worldvolume, this definition loses its generality. Is it that a state consists of the values of all the degrees of freedom at a particular point in the background space? As an example, would we say that a relativistic string is in a particular state if we know the location in the target space of a particular point of its worldsheet?

This would be one possibility, but it would miss the point that eventually, we are interested in the evolution of the system throughout the whole background space (the

mechanics, they are operator-valued functions of the background domain. In this sense, the distinction between classical and quantum mechanics lies in the difference between two paradigms for constructing a mathematical model of a system.

worldsheet in this case) – we are interested in integrating the equations of motion. In the case of a scalar field (a particular target space coordinate) over the worldsheet, with such limited information we would probably not be able to deduce the positions of other worldsheet points. This is why we adopt the following working definition for a system state: a state is a specification of the degrees of freedom of the system over a subset of the background space, together with enough additional information (such as derivatives) to deduce the values of the degrees of freedom at every point of the background space, by means of the Lagrangian. In this way, it is possible to "slice up" the block picture of a fully integrated system into a sequence of states, even for systems that do not have a one-dimensional background space. Essentially, we are saying that a particular state fixes the values of the degrees of freedom at not one but maybe multiple points of the background space, possibly with the values of their derivatives. The purpose of such a definition is that if we know that the system is in a particular state and we know what the equations of motion are, we could in principle deduce what are the other states that a system will go through.

Such a definition is of course valid for classical theories; in the quantum case, it suffices to say that a state specifies not particular values for the degrees of freedom, but probability distributions for each degree of freedom. This information is encoded as a normalised state vector in the Hilbert space of the system.

Finally, we arrive at the problem of what precisely it means to identify two distinct theories. The reason for which states are useful is that experiment always samples a system within a restricted region of the background domain space; it evaluates the degrees of freedom or functions of these (dynamical variables) at a particular time, for example. In other words, the act of measurement helps us pinpoint a particular state of the system. This is a crucial point: we identify the states of a system via physical observables – those dynamical variables that are gauge invariant and that can be measured. If we can match the observables of two different systems such that, for certain states on both sides, the matched observables yield identical values, then, as far as these particular states and observables are concerned, the two systems describe the same physics. Hence, whatever can be described using one theory can equally well be described by the other.

Let us illustrate this idea with an example. First, consider the quantum theory of a relativistic point particle of mass m. The Hilbert space contains eigenstates of the four-momentum operator which we can label as $|E, \vec{p}\rangle$, with E and $\vec{p}$ satisfying the relativistic energy-momentum relation. Since they are eigenstates, each state has a definite energy and momentum – we even label the states by the values of the four-momentum observable. Now take the quantum field theory of a real scalar field of the same mass m. When we construct its spectrum, we find states of the form $a_{\vec{p}}\,|0\rangle$, which are states created from vacuum with definite three-momentum $\vec{p}$ and on-shell energy. We can therefore identify the one-particle states of the quantum scalar field with the states of the relativistic quantum point particle [6, §11.4]. This identification means that whenever we encounter a state with these properties, it doesn't really matter whether the underlying system is a scalar field or a point particle – from the point of view of measuring the observables energy and momentum, the two systems are indistinguishable. In a certain sense, the state – even though we defined it as some subunit of the system – has become independent of the system and has acquired a fundamental character of its own.

Therefore, we can claim that two theories are equivalent if we can find a way to match observables on both sides in a consistent way, because observables are those entities in the mathematical formalism that bring out the physical content of states. This is how the correspondence is formulated in the case of AdS/CFT: correlation functions are matched on both sides, and correlation functions are nothing more than expectation values of quantum dynamical variables. It is also worth noting that it is not necessary for the observables to be the same functions of the degrees of freedom in both theories – the two theories might not even have the same degrees of freedom to begin with. What is important is that once observable $\mathcal{O}_A$ of theory A is matched to observable $\mathcal{O}_B$ of theory B for a particular pair of states, then $\mathcal{O}_A$ and $\mathcal{O}_B$ match for all other states.

There are of course limitations to such an equivalence. First, the states of two distinct systems can be equivalent for some purposes, but not for others: we might decide to measure observables in one theory that have no matched partner in the other. Also, there might be states which are excluded from the correspondence: consider for example the many-particle states in the scalar field theory which do not map to states in the point particle theory. An equivalence therefore has a *scope of validity*, and such situations are outside of its scope.

Let us finish this section by recalling that AdS/CFT is a particular type of correspondence - it is a duality. When two theories are matched against each other, their parameters must also be matched. Dualities relate theories such that when one special parameter is small ($\ll 1$) in one theory, the corresponding parameter is large ($\gg 1$) in the other theory. In the case of AdS/CFT, this parameter is the 't Hooft coupling λ, which on the gravity side is related to the curvature of the background, whereas on the gauge theory side it is the effective coupling when N is taken to be large [7, §8]. Thus, when one theory is weakly coupled – i.e. the correlation functions can be well approximated by a perturbative expansion in the 't Hooft coupling – the other one is strongly coupled, and vice versa.

2.2 The gauge theory side

In this section, we will introduce the preliminaries needed to understand the gauge side of the correspondence and how $\mathcal{N} = 4$ SYM, which is a field theory, arises in a string theory setting. We will briefly discuss conformal symmetry, supersymmetry, and how representations of the superconformal group are classified; we will then discuss a crucial feature of string theory – D-branes – and see how the theory that lives on a special type of D-branes is actually $\mathcal{N} = 4$ SYM.

2.2.1 Conformal symmetry

$\mathcal{N} = 4$ supersymmetric Yang-Mills is a conformal field theory: it is invariant under conformal transformations. Conformal transformations are a generalisation of scale transformations. While a scale transformation corresponds to scaling the metric by a constant factor throughout spacetime, a conformal transformation scales the metric by a factor which varies with spacetime coordinates. More precisely, a conformal transformation is a coordinate transformation $x \to x'(x)$ such that

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \Omega^2(x) g_{\mu\nu}(x), \tag{2.1}$$

which preserves the angles between tangent vectors, but not their norms.

For Minkowski spacetime $\mathbb{R}^{1,3}$, the conformal transformations are divided into 4 types:

- Translations with generators P_μ;
- Lorentz transformations with generators $L_{\mu\nu}$;
- Dilatations, i.e. scaling transformations by a constant factor:
$$x^\mu \to x'^\mu = \lambda x^\mu,$$
generated by an element D in the Lie algebra;
- Special conformal transformations, given by
$$x^\mu \to x'^\mu = \frac{x^\mu + a^\mu x^2}{1 + 2x \cdot a + a^2 x^2},$$
which have generators K_μ.

The Poincaré algebra, involving the generators P_μ and $L_{\mu\nu}$, is a subalgebra of the conformal algebra: indeed, the Poincaré transformations preserve the Minkowski metric, so they satisfy trivially the defining condition (2.1) and the Poincaré group is a subgroup of the conformal group. The commutation relations between the rest of the generators are as follows [8, §2.1.1]:

$$\begin{aligned}
[D, P_\mu] &= -iP_\mu \\
[D, L_{\mu\nu}] &= 0 \\
[L_{\mu\nu}, K_\rho] &= -i(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu) \\
[P_\mu, K_\nu] &= 2i\,(L_{\mu\nu} - \eta_{\mu\nu} D) \\
[D, K_\mu] &= iK_\mu.
\end{aligned} \tag{2.2}$$

It can be shown [8] that the conformal algebra (2.2) is isomorphic to to the algebra of $SO(2,4)$, which is therefore the simply connected component of conformal group in 3+1 dimensions.

As conformal field theories treat different scales the same, the states of the theory cannot be labelled by their mass, as different scales correspond to different energies. Therefore, under a conformal transformation, a state can change its squared mass - the latter is no longer an invariant under the full symmetry group of the theory. Thus, in the analysis of a CFT, the emphasis is placed on operators and their transformation rules, and all states in the theory are massless [9, §4].

2.2.2 Supersymmetry

Supersymmetry (SUSY) is yet another extension of the Poincaré algebra. In an attempt to incorporate internal symmetries into spacetime symmetries, it allows for transformations between objects with different spins, and hence is a symmetry between bosons and fermions. The supersymmetry algebra is a graded Lie algebra: its elements $\mathcal{O}_a$ are assigned a type η_a which can take two values:

$$\eta_a = \begin{cases} 0, & \text{if } \mathcal{O}_a \text{ is bosonic} \\ 1, & \text{if } \mathcal{O}_a \text{ is fermionic} \end{cases} \tag{2.3}$$

where a runs from 1 up to the dimension of the algebra. The algebra is then given by

$$\mathcal{O}_a\mathcal{O}_b - (-1)^{\eta_a\eta_b}\mathcal{O}_b\mathcal{O}_a = i f^c_{ab}\mathcal{O}_c \tag{2.4}$$

where f^c_{ab} are the structure constants. Essentially, combinations with a bosonic generator obey a commutation relation, whereas combinations of two fermionic generators obey an anti-commutation relation.

To extend the Poincaré algebra, which consists solely of bosonic generators, new fermionic generators Q are introduced. They obey relations of the form

$$[L_{\mu\nu}, Q] \sim Q \tag{2.5}$$

and thus form themselves a representation of the Poincaré algebra under the adjoint action. Because of their anti-commuting nature, the spinor representation is chosen, and hence fermionic generators come in a pair $Q_\alpha, \alpha = 1, 2$ that transforms as a spinor. In addition to these, there are generators transforming under the conjugate spinor representation $\bar{Q}_{\dot{\alpha}}$. Finally, we might have more than one set of such generators, in which case we speak of several supersymmetries and the generators are labelled Q^a_α and $\bar{Q}_{\dot{\alpha}a}$, with $a = 1, ..., \mathcal{N}$, where $\mathcal{N}$ is the number of distinct supersymmetries. The supersymmetry algebra is given by he commutation relations of the Poincaré algebra together with the (anti-)commutation relations for the fermionic generators Q, also called the supercharges [10, §2.2.2, §2.3.1]:

$$\begin{aligned} [Q^a_\alpha, L_{\mu\nu}] &= (\sigma_{\mu\nu})_\alpha{}^\beta Q^a_\beta \\ [Q^a_\alpha, P_\mu] &= 0 \\ \{Q^a_\alpha, Q^b_\beta\} &= \epsilon_{\alpha\beta} Z^{ab} \\ \{Q^a_\alpha, \bar{Q}_{\dot{\beta}b}\} &= 2\delta^a{}_b P_\mu (\sigma^\mu)_{\alpha\dot{\beta}} \end{aligned} \tag{2.6}$$

where $\sigma^{\mu\nu}$ are the generators of $SL(2,\mathbb{C})$ given by

$$(\sigma^{\mu\nu})_\alpha{}^\beta \equiv \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)_\alpha{}^\beta$$

and $\sigma^\mu, \bar{\sigma}^\mu$ are the four-vectors of Pauli sigma matrices and the identity,

$$\sigma^\mu \equiv (\mathbb{1}, \sigma^i)$$
$$\bar{\sigma}^\mu \equiv (\mathbb{1}, -\sigma^i).$$

The supercharges corresponding to different supersymmetries can be rotated between each other - this gives rise to an automorphism symmetry group $SU(\mathcal{N})$ of the super-Poincaré algebra, called R-symmetry. Similarly to the conformal symmetry group $SO(2,4)$, this symmetry will play a crucial role in the mapping between the two sides of the AdS/CFT correspondence.

2.2.3 Special operators in superconformal theories

We discussed two extensions of the Poincaré algebra - conformal symmetry and supersymmetry in $\mathbb{R}^{1,3}$. Brought together, they form the superconformal group in 3+1 dimensions, $SU(2,2|4)$. The $SU(2,2)$ part refers to the subgroup $SO(2,4)$ corresponding to conformal transformations - there exists a group homomorphism between the two; the $SU(4)$ part refers to R-symmetry of the supercharges, sometimes denoted as $SU(4)_R$. We are thus led to study the representations of the superconformal group. We present here a brief overview of how they are classified; the reader is referred to [11, §3.3] or [8, §2.1] for more comprehensive treatments.

As mentioned in section 2.2.1, in a conformal field theory, the attention is more focused on the transformation properties of quantum operators and combinations of these, rather than those of the states of the system. Different operators fall into different representations of the superconformal group. There is, however, a one-to-one correspondence between a specific type of operators in a CFT called chiral primary operators, and unitary representations of the superconformal group. There are several intermediate steps before defining these operators.

A local operator in QFT is an operator-valued product of fundamental fields of a system and possibly their derivatives - it is analogous to a dynamical variable in classical mechanics[2]. Operators in a CFT are classified according to their scaling dimension Δ, their eigenvalue with respect to the dilatation generator D. As can be seen from the first and last relations in the conformal algebra (2.2), the generators P_μ and K_μ act as raising and lowering operators with respect to D.

[2]Care must be taken when taking the product of quantum fields at the same spacetime point, as often the expression might exhibit singular behaviour [12].

A non-vanishing local operator $\mathcal{O}(x)$ which satisfies

$$[K_\mu, \mathcal{O}(x)] = 0 \qquad \text{for } x = 0 \tag{2.7}$$

is called a conformal primary operator.

The superconformal algebra introduces new supercharges S^a_α which appear in the commutators of the Poincaré supersymmetries and the special conformal transformations K_μ, and are thus required to close the algebra. An operator $\mathcal{O}(x)$ which in addition satisfies, for all a, α,

$$\begin{aligned} [S^a_\alpha, \mathcal{O}(0)] &= 0 \qquad \text{if } \mathcal{O}(x) \text{ is bosonic} \\ \{S^a_\alpha, \mathcal{O}(0)\} &= 0 \qquad \text{if } \mathcal{O}(x) \text{ is fermionic} \end{aligned} \tag{2.8}$$

is called a superconformal primary operator. Every irreducible representation of the superconformal group contains one superconformal primary operator, which is the one of lowest dimension in the representation. If a superconformal operator is in addition annihilated by at least some of the supercharges Q, then it is called a chiral primary operator. Chiral primary operators fall into different categories according to the number of supercharges Q that annihilate them. Of relevance for AdS/CFT are the so called 1/2-BPS single trace operators, as they are directly mapped to objects on the supergravity side; they are annihilated by 8 supercharges Q.

2.2.4 $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

Having discussed the symmetries of $\mathcal{N} = 4$ SYM and how operators are classified under these symmetries, it is now time to write down the Lagrangian of the theory on the gauge side of the AdS/CFT correspondence. The content of $\mathcal{N} = 4$ SYM was given in section 2.1, and the background space is $\mathbb{R}^{1,3}$. The Lagrangian is

$$\begin{aligned} \mathcal{L} = \mathrm{tr}\Bigg(&-\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\theta_I}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} - \sum_a i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda_a - \sum_i D_\mu \phi^i D^\mu \phi^i \\ &+ \sum_{a,b,i} g C_i^{ab} \lambda_a [\phi^i, \lambda_b] + \sum_{a,b,i} g \bar{C}_{iab} \bar{\lambda}^a [\phi^i, \bar{\lambda}^b] + \frac{g^2}{2} \sum_{i,j} [\phi^i, \phi^j]^2 \Bigg) \end{aligned} \tag{2.9}$$

where $\phi^i, i = 1, ..., 6$ are the six scalar fields, $\lambda^a, a = 1, ..., 4$ are the four left Weyl fermions, A_μ is the gauge field, g is the gauge coupling and θ_I is the instanton angle. The field strength F, its Poincaré dual $\tilde{F}$ and the covariant derivative D_μ are defined as

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \\ \tilde{F}_{\mu\nu} &= \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \\ D_\mu \lambda &= \partial_\mu \lambda + i[A_\mu, \lambda], \end{aligned}$$

and the constants C_i^{ab} and C_{iab} come from the Clifford Dirac matrices for $SO(6)$, isomorphic to $SU(4)$ [4, §3].

This theory is supersymmetric by construction, but also exhibits conformal symmetry. It has 16 real supercharges. One very interesting property of $\mathcal{N} = 4$ SYM is that it is a finite theory, meaning that there are no divergences and the β function identically vanishes at perturbative level.

2.2.5 Elements of string theory: D-branes

In this final subsection, we will explore how $\mathcal{N} = 4$ SYM arises in a string-theoretic setting. For this, we introduce the objects in string theory which are central to the AdS/CFT correspondence: D-branes.

There are two basics sectors of excitations in string theory – open and closed strings. When string theory is quantised, the excited states of these objects can be equivalently described as the excited states of quantum fields permeating the whole or part of the target space. This means that the spectrum of a quantum string theory is identified with the spectra of multiple quantum fields. For example, the graviton is a massless excited state of the closed string, identified with the quantum of a field that lives in the target space. To quantise an open string, however, some specific boundary conditions need to be imposed for its endpoints. This is where D-branes come into the picture: they are multi-dimensional hypersurfaces embedded in the target space where open strings can end. Such a hypersurface of $p + 1$ dimensions is referred to as a Dp-brane.

A crucial fact about D-branes is that they are dynamical objects of their own: the excitations of open strings with endpoints constrained to the D-brane can be interpreted as fluctuations of the D-brane itself, by identification with the fluctuations of quantum fields constrained to live on the D-brane. The massless excitations of open strings on a Dp-brane can be identified with the spectrum of an Abelian gauge field $A_\mu, \mu = 0, ..., p$ and scalar fields $\phi^i, i = 1, ..., 9 - p$, all of which live on the $p + 1$ dimensional worldvolume of the brane [13, §4.2.2]. The scalar fields can be interpreted as fluctuations of the brane in 9+1 dimensional spacetime in the transverse directions to the brane; they are seen to couple to the gauge field A_μ through an action called the Dirac-Born-Infeld (DBI) action [9, §3.2, §7.6].

In a similar but rather more evolved way, when we consider the excitations of open strings on a D3-brane in Type IIB superstring theory, we find that the low-energy massless excitations are described by a $\mathcal{N} = 4$ SYM $U(1)$ gauge theory. When N coincident D3-branes are considered, then the theory is enhanced to a $\mathcal{N} = 4$ SYM with gauge group $U(N)$ [9, §7.7]. This first interpretation of D3-branes is what makes it possible for a gauge theory to emerge out of a string setting. We will see in the next sections that there is a second interpretation for D3-branes which makes possible the identification of $\mathcal{N} = 4$ SYM with the string side of the AdS/CFT correspondence.

2.3 The supergravity side

This section is devoted to describing the geometry of Anti-de-Sitter place, the target space for the string theory side of the correspondence. At the end of this section, we introduce the second interpretation of D-branes that stems from supergravity.

2.3.1 Anti-de-Sitter space

Let us now describe in detail the geometry of the background space on the gravity side of the correspondence: 5-dimensional Anti-de-Sitter space (AdS_5). AdS_5 is the maximally symmetric vacuum solution of the Einstein equations in 5 dimensions with a negative cosmological constant Λ. A maximally symmetric space is one that has the same number of symmetries as Euclidean space of the same dimension. Other maximally symmetric solutions of the vacuum Einstein equations in 5 dimensions include the 5-sphere S^5, the hyperboloid H^5 and 5-dimensional de Sitter space, dS_5. The sphere and dS have constant positive curvature $\mathcal{R}$, whereas for the hyperboloid and AdS the curvature is constant and negative; all four can be embedded in 6-dimensional flat space with an appropriate signature. The metrics of the sphere and the hyperboloid have a Euclidean signature, whereas dS and AdS are Lorentzian surfaces. Thus, to define AdS_5, consider $\mathbb{R}^{2,4}$ with metric

$$ds^2 = -\,dX_0^2 - dX_5^2 + dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2$$

and embed AdS_5 as the surface given by the equation

$$X_0^2 + X_5^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2 = R^2. \tag{2.10}$$

Now consider a set of new coordinates (τ, ρ, y_i), $i \in 1, ..., 4$, given by

$$\begin{aligned} X_0 &= R\cosh\rho\cos\tau \\ X_5 &= R\cosh\rho\sin\tau \\ X_i &= R(\sinh\rho)y_i \end{aligned} \tag{2.11}$$

with the condition

$$\sum_{i=1}^{4} y_i^2 = 1 \tag{2.12}$$

Substituting for the new coordinates readily solves equation (2.10). Note that only five of the new coordinates are independent due to condition (2.12), thus they are a parametrisation of AdS_5. The metric then reads

$$ds^2 = R^2(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2) \tag{2.13}$$

where $d\Omega_3$ is the line element on the unit 3-sphere. The coordinates (τ, ρ, y_i) are global as they cover AdS_5 completely in the ranges $\rho \in [0; +\infty[$ and $\tau \in [0; 2\pi]$. To avoid closed timelike curves arising from the periodicity of τ, we will always take the universal cover that can be "wrapped" around this space, where $\tau \in \mathbb{R}$ and no identifications are made. It is this covering space that we will refer to as AdS_5.

Nevertheless, another set of coordinates for AdS_5 will be convenient for the treatment of AdS/CFT. These are Poincaré coordinates $(u, t, \vec{x})$ [14, §1.2], and they are obtained by setting

$$\begin{aligned}
X_0 &= \frac{1}{2u}\left(1 + u^2(R^2 + \vec{x}^2 - t^2)\right) \\
X_a &= Rux_a \qquad\qquad\qquad\qquad a = 1, 2, 3 \\
X_4 &= \frac{1}{2u}\left(1 - u^2(R^2 - \vec{x}^2 + t^2)\right) \\
\text{‘}X_5 &= Rut
\end{aligned} \tag{2.14}$$

which casts the metric in the form

$$ds^2 = R^2\left(\frac{du^2}{u^2} + u^2\eta_{\mu\nu}dx^\mu dx^\nu\right) \tag{2.15}$$

where η is the Minkowski metric with signature $(-, +, +, +)$. Note that these coordinates only cover half of AdS_5: the domain of the "radial" coordinate u is $]0; +\infty[$ and there is a coordinate singularity at $u = 0$. Because the Killing vector ∂_t has null norm at $u = 0$, the latter is also a horizon.

From the form of the metric (2.15), we see that 5-dimensional Anti-de-Sitter space can be visualised as a foliation of many 4-dimensional Minkowski spacetimes living at the different values of u. To investigate the behaviour at large u, we take the conformally equivalent metric

$$d\tilde{s}^2 = \frac{ds^2}{u^2} = R^2\left(\frac{du^2}{u^4} + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

and note that it has a boundary at $u \to +\infty$, which is simply $\mathbb{R}^{1,3}$. Thus, we say that the conformal boundary of AdS_5 is $\mathbb{R}^{1,3}$. The claim of AdS/CFT conjecture is a correspondence between a string theory that lives in the bulk of $AdS_5 \times S^5$ and a field theory living on precisely this boundary.

Before concluding this section, let us rescale the radial coordinate for later use. Writing

$$u = \frac{r}{R^2},$$

the metric now reads

$$ds^2 = \frac{R^2}{r^2}dr^2 + \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu \tag{2.16}$$

following from direct substitution in equation (2.15).

2.3.2 Supergravity

Supergravity is a theory of local supersymmetry that incorporates SUSY into General Relativity. Of particular interest to the present discussion are objects called p-branes, which are solitonic solutions to the equations of supergravity [7, §5]. They are in fact black hole solutions, similar to the Schwarzschild black hole, that extend in p spatial dimensions. Furthermore, a p-brane is an extremal solution, meaning that it has a charge equal to its mass. One of the major insights in string theory consisted in identifying p-branes, solitonic solutions of supergravity, with Dp-branes, hypersurfaces in the target space where open strings can end. This duality of the branes is at the core of the AdS/CFT correspondence.

2.4 Formulation of the correspondence

In the previous two sections, we explored some of the features of the two theories that are connected by the AdS/CFT correspondence. We are now ready to see how the link is made between them. As outlined in section 2.1, we are looking for a way to match the observables of these two theories – in this way we can establish that the two mathematical models describe the same physics.

The setup in which the correspondence is realised is flat 9+1 dimensional Minkowski spacetime $\mathbb{R}^{1,9}$ in which N D3 branes are immersed and whose positions coincide. The duality arises as a result of the dual interpretation of D3 branes – we can view them either as hypersurfaces in $\mathbb{R}^{1,9}$ where open strings can end, or as solitonic solutions of Type IIB supergravity which bend the original flat geometry. Hence, according to the first interpretation we have a string theory in flat space in which we will consider the excitations of open and closed strings, whereas according to the second interpretation we have a string theory in curved space with excitations of closed strings only.

Maldacena's original conjecture [3] is not, however, a correspondence between these two full string theories on different backgrounds – rather, it relates parts of these theories in a particular low-energy limit, also called the Maldacena limit [11, 4]. The Maldacena limit consists in taking $\alpha' \to 0$, while at the same time keeping g_s, N, and all other dimensionless quantities fixed. To sketch the general idea, in this limit each theory breaks up into two independent pieces – two systems that do not interact with each other. As we will see, the second pieces of both original theories turn out to be identical. In other words, if string theory A splits into A1 and A2, and string theory B splits into B1 and B2, we find that A2 is the same as B2. The claim of the AdS/CFT correspondence then consists in identifying the first pieces of theories A and B, namely A1 and B1 (which are not the same at all).

Let us see exactly what these pieces are. Coming back to the first interpretation of D3 branes as the allowed locations for endpoints of open strings, we will consider only massless excitations of the latter. This is justified in the low-energy limit, where the massive ones would be too high in the spectrum to be reached with the available amount of energy in the system. Thus, we will consider an effective action S_{eff} for massless string modes:

the massive modes have been integrated out [8, §3.1]. Such an action consists of three parts: the action of the open strings with endpoints on the D3 brane stack, S_{branes}, the action of the closed strings throughout the whole spacetime, S_{bulk}, and the action term corresponding to interactions between the two, S_{int}:

$$S_{\text{eff}} = S_{\text{branes}} + S_{\text{bulk}} + S_{\text{int}} \tag{2.17}$$

Since we are considering massless modes only, the bulk action of the closed string modes can be written as a sum of a free propagation part and a massless interaction part. Writing the metric as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, where κ^2 is the gravitational coupling, proportional to Newton's constant, we can write schematically [11, §2.2],

$$S_{\text{bulk}} = \frac{1}{2\kappa^2}\int d^{10}x \left[\sqrt{|g|}\mathcal{R} + \mathcal{O}\left(\mathcal{R}^2\right)\right] \sim \int d^{10}x \left[(\partial h)^2 + \kappa(\partial h)^2 h + \cdots\right] \tag{2.18}$$

where the contraction of the indeces between the perturbation of the metric h and the partial derivative is non trivial and omitted here – see [9, §2.3.2] for the precise contraction prescription. But κ is proportional to $g_s\alpha'^2$, hence $\kappa \to 0$ as $\alpha' \to 0$. Hence in the Maldacena limit, the bulk action reduces to free supergravity, the low-energy limit of a string theory with massless excitations.

The other terms in S_{eff} also simplify: as discussed in section 2.2.5, open strings living on a stack of N D3 branes give rise to a $\mathcal{N} = 4$ SYM theory with gauge group $U(N)$. Hence the brane part S_{branes} of the effective action contains the $\mathcal{N} = 4$ SYM action and some higher order terms proportional to positive powers of α' which all vanish as $\alpha' \to 0$. Finally, the interaction term S_{int} is itself proportional to κ, and hence drops out in the same limit.

We therefore see that

$$S_{\text{eff}} \xrightarrow{\alpha' \to 0} S_{\mathcal{N}=4\ \text{SYM}} + S_{\text{supergravity}} \tag{2.19}$$

meaning that in the Maldacena limit, the string theory of open and closed strings in $\mathbb{R}^{1,9}$ with N D3 branes reduces to two decoupled theories, 4-dimensional $\mathcal{N} = 4$ SYM living on the D3 branes and 10-dimensional free supergravity.

Let us now consider the stack of N D3 branes from the second point of view, namely as solitonic solutions to Type IIB supergravity. As discussed in section 2.3.2, this means that they have mass and charge – they act as sources under a five-form field strength and they deform spacetime into a solution of the Einstein equations coupled to this field strength. For 3+1 dimensional branes, such a solution is given by the following metric [8, §3.1]:

$$ds^2 = H^{-1/2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H^{1/2}(dr^2 + r^2 d\Omega_5^2) \tag{2.20}$$

where

$$H \equiv 1 + \frac{R^4}{r^4},$$

R is determined by the number N of D3 branes that we are considering:

$$R^4 = 4\pi g_s \alpha'^2 N,$$

and $d\Omega_5^2$ is the metric on the unit 5-sphere. The D3 branes sit at the origin and extend in the t, x_1, x_2 and x_3 directions.

Before taking the Maldacena limit, let us explore the small and large r behaviour of the system. For $r \gg R$, the factor $H \to 1$ and the metric (2.20) reduces to the flat 9+1 dimensional Minkowski metric. The geometry is therefore asymptotically flat. In the opposite limit, $r \ll R$, we have $H \to \frac{R^4}{r^4}$ and the metric looks like

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2 \tag{2.21}$$

in which we recognise the metric (2.16) of Anti-de-Sitter space in 5 dimensions and the round metric of the 5-sphere, both with radius R. This is the $AdS_5 \times S^5$ geometry.

As an object is brought closer to $r = 0$, its energy as measured by an observer at infinity is redshifted [8]. Thus excitations of closed strings which are close to the D3 branes are seen as low-energy excitations by an observer at $r = \infty$. Furthermore, when we take the Maldacena limit, near-horizon excitations and excitations at large radius r (in the bulk) decouple, and we are left with independent theories once again. The theory in the bulk turns out to be again the low-energy approximation of string theory with D3 branes – type IIB supergravity, whereas the theory in the near-horizon limit is Type IIB superstring theory on $AdS_5 \times S^5$.

We are thus led to the original conjecture of Maldacena, namely that type IIB superstring theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N} = 4$ SYM on the conformal boundary of AdS_5, $\mathbb{R}^{1,3}$.

Before moving on to a more precise mapping between the observables of the two theories, let us mention the three forms of the correspondence which are usually discussed. In the strongest form, the conjecture is assumed to be valid for all values of N and coupling $g_s = g_{YM}^2$. In the moderate form, the conjecture is assumed to be valid after the the 't Hooft limit has been taken; this is defined as taking $N \to +\infty$ but keeping the 't Hooft coupling $\lambda \equiv g_s N = g_{YM}^2$ constant. Finally, in the weakest form of the correspondence, it is only assumed to be valid for large values of the 't Hooft coupling, $\lambda \gg 1$.

2.4.1 The field-operator map between the two theories

Although the correspondence is assumed to be valid in its strongest form, we will primarily discuss its weak form, where the gauge theory (for which the effective coupling is λ), is strongly coupled, whereas the string theory reduces to its low-energy Type IIB supergravity approximation. We will attempt in this section to motivate further a correspondence between the observables in the two theories by exploring the symmetries on each side.

As discussed in section 2.3.1, the isometry group of AdS_5 is $SO(2,4)$, whereas the isometry group of S^5 is simply $SO(6)$. On the other hand, the symmetry group of $\mathcal{N}=4$ SYM is the superconformal group $SU(2,2|4)$, which contains $SO(2,4)$ (the group of conformal transformations) and $SU(4)_R \simeq SO(6)$ (the R-symmetry group of the supercharges). Thus we see that the global symmetry groups on both sides match, meaning that it is possible to identify the observables on both sides in a sensible way that respects the transformation rules. Note that the gauge group of the gauge theory does not have an equivalent on the gravity side - thus any matched observable must be gauge-invariant. To proceed, we would need to match the particular representations of the symmetry groups. Here, we will limit the discussion to the type of objects that are matched on both sides.

As we saw in section 2.2.3, representations of the superconformal algebra are associated with chiral primary operators of the gauge theory. On the gravity side, supergravity fields that have been dimensionally reduced on S^5 by Kaluza-Klein reduction fall into representations of the isometry group of $AdS_5 \times S^5$. Hence operators on the gauge side should correspond to fields on the gravity side. In particular, we state without proof that the single trace operators mentioned in section 2.2.3 correspond to the canonical fields on the supergravity side [4, §5.6].

2.4.2 Correspondence at the level of correlation functions

We conclude this chapter with the precise prescription for computing observables on one side of the correspondence using the other. This connection between the correlation functions of the two theories was first given by Witten [15]; it is formulated at the level of the partition functions, from which correlation functions can be obtained by differentiation with respect to the source which is set it to zero at the end. Let the partition function of a field φ in the string theory on $AdS_5 \times S^5$ be $\mathcal{Z}_\varphi^{string}$, and let the value of the field on the boundary of $AdS_5 \times S^5$, $\mathbb{R}^{1,3}$, be φ_0. Then to obtain the partition function of the operator in the CFT corresponding to that particular field, the boundary value φ_0 is used as a source to which the operator on $\mathbb{R}^{1,3}$ couples. Thus, denoting the partition function of the operator $\mathcal{O}$ coupled to the source φ_0 as $\mathcal{Z}_\mathcal{O}[\varphi_0]$, we have

$$\mathcal{Z}_\mathcal{O}[\varphi_0] = \int \mathcal{D}A\mathcal{D}\phi\mathcal{D}\lambda e^{-S_{SYM}+\int d^4x\mathcal{O}(x)\varphi_0(x)} \tag{2.22}$$

where we have used Euclidean space rather than Minkowksi space.

The precise formulation of the AdS/CFT correspondence consists in saying that these two partition functions are equal:

$$\mathcal{Z}_\varphi^{string}\big|_{\varphi=\varphi_0 \text{ on boundary}} = \mathcal{Z}_\mathcal{O}[\varphi_0] \tag{2.23}$$

where the subscript indicates the field for the partition function and the argument indicates the source.

In the supergravity limit, where $g_s \to 0$ and $\alpha' \to 0$ and quantum corrections are small, the stationary phase approximation can be taken and the string partition function reduces to the contribution of the classical path:

$$\mathcal{Z}_{\varphi}^{string}\big|_{\varphi=\varphi_0 \text{ on boundary}} \approx e^{-S_{SUGRA}[\varphi]}\big|_{\varphi=\varphi_0 \text{ on boundary}} \tag{2.24}$$

and thus we have

$$\int \mathcal{D}A\mathcal{D}\phi\mathcal{D}\lambda e^{-S_{SYM}+\int d^4x\mathcal{O}(x)\varphi_0(x)} = e^{-S_{SUGRA}[\varphi]}\big|_{\varphi=\varphi_0 \text{ on boundary}} \tag{2.25}$$

which is the mathematical expression for the weak form of the correspondence.

Chapter 3

Applications of AdS/CFT to QCD

After having explored the original example of AdS/CFT in some detail, we are going to briefly sketch in this chapter how the correspondence is generalised to field theories with properties more similar to those of QCD. In doing so, we are entering the broader realm of the more general gauge/gravity or gauge/string dualities. The important features that we will focus on are the addition of matter degrees of freedom and the breaking of conformal invariance on the field theory side.

3.1 Addition of quarks (flavours)

The degrees of freedom in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory all form part of a gauge supermultiplet: there are no fields that transform in the fundamental representation of the gauge group $SU(N)$, whereas quarks in QCD are precisely such fields. The basic method for including quarks in a gravity dual is to add another type of branes to the theory, so that open strings can have one end on the D3 branes and the other on the new branes. This generates degrees of freedom in the fundamental representation [1, §3]. On the other hand, meson states correspond to strings ending with both ends on the new flavour branes. The simplest approach uses N_f D7 branes as flavour branes, originally devised by [16]. The corresponding field theory contains the gauge supermultiplet of $\mathcal{N} = 4$ SYM and a N_f hypermultiplets transforming in the fundamental representation in a $\mathcal{N} = 2$ U(N) gauge theory.

The idea has been applied to different approximations. In the "quenched", or "probe brane" approximation, only a small number of additional flavour branes are considered, whereas the number of D3 branes is taken to infinity. This, however, results in separations of masses of meson states different from those in QCD [1]. On the other hand, in the backreaction approximation, the number of flavour D7 branes is taken to be comparable to that of the D3 branes and their effect on the geometry is no longer neglected. The computations in this modified geometry tend to be more difficult and progress towards

QCD phenomena is limited.

There exist many other variants of the basic idea of introducing another type of brane, usually amounting to selecting the type of brane used. The largest brane that can be used without intersecting the D3 branes is a D8 brane; one of the benefits of this approach is that it leads to chiral fermions [17, §3.2]. A recent review of three different models, each incorporating a different set of flavour branes, can be found in [18].

3.2 Incorporation of confinement (breaking conformal invariance)

Confinement is expressed by a potential between quarks that increases linearly with separation. This corresponds to a Wilson loop operator exhibiting an area law at large distances. Wilson loops on the gauge side are dual to string worldsheets ending with their boundaries on the Wilson loops. To compute the energy between the quarks, one computes the Nambu-Goto action of the string worldsheet. For AdS space, which is dual to a conformal field theory, this scales as the inverse of the length on the boundary, signalling a lack of confinement. If the geometry is altered by imposing, for example, a cut-off at a fixed radial coordinate – or by inserting a black hole – a confining potential can be achieved [1, §2.5]. More detailed information about confining theories can be found in [17] and [19].

Chapter 4

Conclusion

In this dissertation, we have presented an overview of the internal workings of the AdS/CFT correspondence. After a general discussion of how a correspondence between any two theories would be established, we investigated the relevant properties of the two theories connected by the correspondence: $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on the gauge side and Type IIB supergravity on the string theory side. Namely, we explored the symmetries of $\mathcal{N} = 4$ SYM and we found that they match the isometries of the background space for supergravity, $AdS_5 \times S^5$. Furthermore, we presented a construction of a stack of D3 branes in flat 9+1 dimensional spacetime, and argued that the system can be equivalently described by two different descriptions due to the dual nature of D-branes. These descriptions were seen to break each into two decoupled theories, the second of which turned out to be the same for both cases. This and the previous symmetry argument led us to the speculation that the gauge theory and the gravity theory might be identified. Finally, we presented the established prescription through which correlation functions on both sides are related to each other, thus identifying the observables of the two theories, and made some remarks about how the duality might be generalised to confining theories and theories with flavour degrees of freedom in the last chapter.

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